Space-Frequency Coding Reduces the Collision Rate in FH-OFDMA

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***Abstract*—A frequency-hopping (FH) space-frequency (SF)-coded clustered orthogonal frequency division multiple access (OFDMA) system is considered. It is shown that SF coding reduces the number of collisions in OFDMA with FH provided that the subcarrier clusters are formed randomly at each hopping instant. Analytical expressions for the collision rate, which is defined as the expected number of collisions per symbol, are derived. Bit error rate (BER) computations via simulations in a frequency-selective fading channel corroborate the theoretical findings for SF-coded FH-OFDMA.**

***Index Terms*—Clustered orthogonal frequency-division multiplexing (OFDM), frequency hopping (FH), orthogonal frequency division multiple access (OFDMA), space-frequency (SF) coding.**

I. Introduction

Dividing the available subcarrier set in orthogonal frequency-division multiplexing (OFDM) among many users gives rise to orthogonal frequency division multiple access (OFDMA). Despite its strength against frequency selectivity, an OFDMA system equipped with transmit (TX) diversity involving space-frequency (SF) coding [2], [8] may occasionally face channel nulls and lose subcarriers to fading. On the other hand, static frequency allocation may result in communications over a channel disturbed by high interference. One additional source of diversity that can inject resistance to fading and interference into OFDMA is frequency (subcarrier) hopping [3], [11].

A generalized frequency-hopping OFDMA (FH-OFDMA) system can eliminate multiuser interference (MUI), as well as narrow-band and partial-band interference, by careful code design [11]. Dynamic FH, where the frequency-hop patterns are adaptively changed to mitigate the time-varying negative channel effects, offer higher user capacity [3], [5]. However, it is usually not desirable for some systems such as ad hoc networks to have coordinated hopping sequence assignment to mobiles.

In this paper, clustered OFDM, which provides a multiple access paradigm by assigning subcarrier clusters to different users [4], is considered. SF coding (SFC) and random FH are integrated into the clustering mechanism, and the system is evaluated from a medium-access control perspective. Random hopping implies MUI in the form of occasional symbol collisions, whose detection and recovery is mandatory for efficient transmission of information. It is proved that SF coding reduces MUI if the subcarriers are clustered independently.

II. SF-Coded FH-OFDMA

Suppose that the N subcarriers of an FH-OFDMA system are partitioned into N/M clusters, with M subcarriers per cluster. Data symbols, drawn from a complex, nonorthogonal constellation, are converted from serial to parallel with block size M. Let $X_k(i,n) := X(i, nM+k)$, $k = 0, 1, \ldots, M-1$, denote the kth symbol of the nth data block generated by the ith user. Then, $\mathbf{x}(i,n) = [X_0(i,n) \;\; X_1(i,n) \;\; \cdots \;\; X_{M-1}(i,n)]^{\mathrm{T}}$ represents the ith user's nth data block, and T is the transpose. Each symbol that makes up the block is transmitted over a distinct subcarrier.

The application of SF block coding generates the two outputs

$$\mathbf{x}_1(i,n) = [X_0(i,n) \; - X_1^*(i,n) \; \cdots \; X_{M-2}(i,n) \; - X_{M-1}^*(i,n)]^{\mathrm{T}} \tag{1}$$

$$\mathbf{x}_2(i,n) = [X_1(i,n) \; X_0^*(i,n) \; \cdots \; X_{M-1}(i,n) \; X_{M-2}^*(i,n)]^{\mathrm{T}} \tag{2}$$

which are defined as in the Alamouti scheme [1].

Every user transmits over M subcarriers, which are assigned or selected in a random manner. Let $C_i(n) = \{f_{i,n,0}, f_{i,n,1}, \ldots, f_{i,n,M-1}\}$ be the ordered set of subcarriers that make up the cluster over which user i transmits its nth data block. Thus, $C_i(n) \subset \{1, \mathrm{e}^{j2\pi/N}, \ldots, \mathrm{e}^{j2\pi(N-1)/N}\}$ has cardinality M for all i and n. The transmission subcarriers in $C_i(n)$ may or may not be contiguous depending on the SF decoding algorithm, and they are altered for all i and n as dictated by the particular random hopping mechanism.

Inverse discrete Fourier transform (IDFT) is applied, and a cyclic prefix (CP) is appended to both $\mathbf{x}_1(i,n)$ and $\mathbf{x}_2(i,n)$ before transmission. The IDFT can be implemented through an N-point inverse fast Fourier transform over the N subcarriers by appropriately zero padding $\mathbf{x}_\ell(i,n)$, $\ell = 1, 2$, to make it an $N \times 1$ vector. The two OFDM blocks containing CP are transmitted through spatially separated or cross-polarized antennas.

The DFT of the channel impulse response observed by user i's TX antenna ℓ generates the transfer functions $\{H_{\ell,0}, H_{\ell,1}, \ldots, H_{\ell,M-1}\}$ at the M subcarriers for $\ell = 1, 2$. The channel gains are assumed to be known and constant for the duration of one OFDM block.

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At the receiver, the CP is removed and DFT is applied so that the channel outputs

$$\begin{bmatrix} Y_m \\ Y_{m+1}^* \end{bmatrix} = \mathbf{H}_m \begin{bmatrix} X_m \\ X_{m+1} \end{bmatrix} + \begin{bmatrix} Z_m \\ Z_{m+1}^* \end{bmatrix}, \quad m = 0, 2, 4, \ldots, M-2$$

where the (i, n) designation is dropped for notational simplicity, and

$$\mathbf{H}_m = \begin{bmatrix} H_{1,m} & H_{2,m} \\ H_{2,m+1}^* & -H_{1,m+1}^* \end{bmatrix}, \quad m = 0, 2, 4, \ldots, M-2$$

are obtained with Z_m representing the aggregate of additive white noise as well as possible MUI to the desired user.

Let $\mathbf{D}_m$ be the SF decoder and $\hat{X}$ denote the estimate of X so that

$$\begin{bmatrix} \hat{X}_m \\ \hat{X}_{m+1} \end{bmatrix} = \mathbf{D}_m \begin{bmatrix} Y_m \\ Y_{m+1}^* \end{bmatrix}, \quad m = 0, 2, 4, \ldots, M-2.$$

Then, $\mathbf{D}_m = \mathbf{H}_m^{\mathrm{H}}$, where H denotes the Hermitian, is the Alamouti scheme, which stipulates that adjacent subcarriers be used to avoid intersymbol interference [8]. The other alternative is to opt for the zero-forcing decoder $\mathbf{D}_m = \mathbf{G}_m$, which is defined as [9]

$$\mathbf{G}_m = \begin{bmatrix} H_{1,m}^* & H_{2,m+1} \frac{\mathrm{e}^{j(\Delta\phi_1 - \Delta\phi_2)}}{\beta_1 \beta_2} \\ H_{2,m}^* \beta_1 \beta_2 \mathrm{e}^{j(\Delta\phi_1 - \Delta\phi_2)} & -H_{1,m+1} \end{bmatrix}, \quad m = 0, 2, 4, \ldots, M-2 \quad (3)$$

where $\beta_1 \mathrm{e}^{j\Delta\phi_1}$ and $\beta_2 \mathrm{e}^{j\Delta\phi_2}$ stand for the relative gain and phase difference between the channel transfer functions of subcarriers $f_{i,n,m}$ and $f_{i,n,m+1}$ for antennas 1 and 2, respectively. The subcarrier selection is completely arbitrary.

III. Hopping Strategies and Collision Analysis

Let U denote the number of active mobiles transmitting simultaneously. Three assumptions are made in the ensuing analysis.

- (AS1) All symbols involved in a subcarrier collision are lost.
- (AS2) Collisions occur only between the symbols of two users.
- (AS3) Channel nulls are absent so that information loss occurs because of collisions only.

Thus, symbol captures are ignored. Because higher order collisions are extremely rare events, (AS2) is plausible. Lastly, noise and fading are neglected in (AS3) to isolate the impact of collisions. In Section IV, (AS1)–(AS3) will be lifted and simulations will take into account all sources of error.

There are two hopping strategies: cluster hopping (CH) and independent hopping (IH).

A. Cluster Hopping

Subcarriers are partitioned into distinct hopping clusters, and each cluster is selected randomly and independently by each user. Thus, hopping takes place among clusters of subcarriers. That is, $C_i(n) \in \mathcal{C}$, where $\mathcal{C} = \{C_1, C_2, \ldots, C_{|\mathcal{C}|}\}$ is the set of predetermined hopping clusters such that the cardinality $|\mathcal{C}| = N/M$, and $C_k \cap C_\ell = \emptyset, \forall k \neq \ell, C_k, C_\ell \in \mathcal{C}$. Users hop among the members of $\mathcal{C}$.

Collisions occur among clusters, and hence, across all the involved OFDM subcarriers. Each user should transmit on a different cluster to avoid collisions. For that reason, SF coding makes no difference for CH as far as collision avoidance is concerned. The expected number of symbol losses per cluster collision is [7]

$$E_{\mathrm{c}}(\mathcal{C}, M) = M \left[1 - \left(1 - \frac{1}{\frac{N}{M}} \right)^{U-1} \right] \quad (4)$$

for CH. Equation (4) indicates that more collisions are expected with increasing number of mobiles. For a constant N in CH, with more subcarriers per user, the cardinality of the hopping set $\mathcal{C}$ decreases and it becomes more likely that users hop to the same clusters and collide. Dividing (4) by M gives the collision rate, defined as the expected number of collisions per symbol, which is inversely proportional to $|\mathcal{C}| = N/M$. However, a large M implies greater symbol loss per cluster collision, regardless of the value of N.

B. Independent Hopping

Each OFDM subcarrier in a cluster is chosen completely randomly and independently of the available subcarriers and other clusters. At any instant, the clusters may be made up of any combination of M subcarriers. Thus, $C_i(n) \in \tilde{\mathcal{C}}$, where $\tilde{\mathcal{C}} = \{\tilde{C}_1, \tilde{C}_2, \ldots, \tilde{C}_{|\tilde{\mathcal{C}}|}\}$, with $|\tilde{\mathcal{C}}| = \binom{N}{M}$, and the members of $\tilde{\mathcal{C}}$ may share subcarriers.

1) IH Without Diversity: Suppose that the FH-OFDMA system employs IH over a single antenna. We first determine the expected number of symbols involved in a collision for $M = 2$ and then generalize the result for any number of subcarriers per user.

Since the OFDM subcarriers of each user hop separately and randomly, they may or may not collide, independent of the other subcarriers. Define $p_M(x)$ to be the probability that x symbols collide given that there are M subcarriers per user. The probability that both subcarriers are collision-free is

$$p_2(0) = \left[\frac{(N-2)(N-3)}{N(N-1)} \right]^{U-1}. \quad (5)$$

Similarly, the probability of just one collision is

$$p_2(1) = 2 \left[\frac{N-2}{N} \right]^{U-1} \left[1 - \left(\frac{N-3}{N-1} \right)^{U-1} \right]. \quad (6)$$

By invoking (AS2), (5) and (6) yield the probability of collision on both channels, $p_2(2)$.

$$p_2(2) = 1 - \left[\frac{(N-2)(N-3)}{N(N-1)} \right]^{U-1} - 2 \left[\frac{N-2}{N} \right]^{U-1} \left[1 - \left(\frac{N-3}{N-1} \right)^{U-1} \right]. \quad (7)$$

The expected number of symbols lost when a cluster collision occurs with IH and two subcarriers per mobile, $E_c(\tilde{\mathcal{C}}, 2)$, is

$$E_c(\tilde{\mathcal{C}}, 2) = 2p_2(2) + p_2(1) = 2\left[1 - \left(1 - \frac{1}{\frac{N}{2}}\right)^{U-1}\right]. \quad (8)$$

Equation (8) turns out to be the same as (4) when the latter is evaluated for $M = 2$. Along similar lines, the collision performance of IH without TX diversity for any M is next.

Lemma 1: Assume that each symbol-collision event is independent of the other symbols' collisions. Then, with IH and no TX diversity, the expected number of symbols lost to a cluster collision, $E_c(\tilde{\mathcal{C}}, M)$, is

$$E_c(\tilde{\mathcal{C}}, M) = \sum_{x=1}^{M} x p_M(x) \quad (9)$$

where

$$p_M(x) = \binom{M}{x}\left[1 - \left(\frac{N - 2M + x}{N - M + x}\right)^{U-1}\right]^x \times \left[\prod_{y=0}^{M-1} \frac{N - M + x - y}{N - y}\right]^{U-1}.$$

The approximation error caused by the independence assumption between the symbol collisions in Lemma 1 is less than 1% provided that the number of users exceeds 3 and the number of subcarriers per user is greater than 2. Numerical evaluations indicate that regardless of the choice of parameters, the expectations in (4) and (9) are approximately equal [6]. Therefore, choosing IH over CH brings no advantage when there is no diversity.

2) IH With TX Diversity: Consider a system with two TX antennas per user so that there are $M/2$ subcarrier pairs for SF coding in total. With antenna diversity available at the transmitter, because each symbol is sent on two distinct subcarrier frequencies, there is a possibility that a symbol colliding in one band will not collide in the other band. In the ensuing derivations, the following is also assumed.

(AS4) A symbol is not lost unless both subcarriers that take part in SF coding face collisions.

Because SF decoders linearly combine the successive receiver inputs, symbols are at a risk even if one of the subcarriers is collision-free. However, the signal-to-interference ratio (SIR) with IH and single subcarrier collision is expected to be considerably higher than the SIR with CH and double collision, and (AS4) is thus justifiable, even though it may not always be realistic.

Depending on the SF decoding, there are two IH systems where (AS4) works.

1) Diversity System 1 (DS1): $\mathbf{D_m} = \mathbf{G_m}$, and the IH clusters are as in Section III-B.
2) Diversity System 2 (DS2): $\mathbf{D_m} = \mathbf{H_m^H}$, but the IH clusters must then consist of pairs of contiguous subcarriers. The number of clusters is reduced to $|\tilde{\mathcal{C}}| = \binom{N-1}{M/2}$, which implies an increased likelihood of collisions.

For DS1 with $M = 2$, the expected number of symbols lost per cluster collision, $E_c^{\mathrm{DS1}}(\tilde{\mathcal{C}}, 2)$, can be calculated as

$$\begin{aligned} E_c^{\mathrm{DS1}}(\tilde{\mathcal{C}}, 2) &= 2p_2(2) \\ &= 2\left\{1 - \left[\frac{N-2}{N}\right]^{U-1}\left[2 - \left(\frac{N-3}{N-1}\right)^{U-1}\right]\right\} \end{aligned} \quad (10)$$

with $p_2(2)$ as in (7). Let $p_{M,L}(x)$ be the probability that x collisions occur in an SFC subcarrier pair given that each user is allocated M subcarriers and L out of the N subcarriers are available for potential interferers (some will be collision-free). The following result generalizes (10).

Lemma 2: Consider an SF-coded FH-OFDMA system with DS1. The expected number of symbols lost in a cluster collision, $E_c^{\mathrm{DS1}}(\tilde{\mathcal{C}}, M)$, is

$$\begin{aligned} E_c^{\mathrm{DS1}}&(\tilde{\mathcal{C}}, M) \\ &= \sum_{x=1}^{\frac{M}{2}} 2x\binom{\frac{M}{2}}{x}\Bigg\{\sum_{y=1}^{\frac{M}{2}-x}\Bigg[\binom{\frac{M}{2}-x}{y}\left[p_{M,N-2y}(1)\right]^{\frac{M}{2}-x-y} \\ &\qquad \times \left[p_{M,N-y-\frac{M}{2}+x}(2)\right]^x \\ &\qquad \times \prod_{z=0}^{y-1} p_{M,N-2z}(0)\Bigg] \\ &\qquad + \left[p_{M,N}(1)\right]^{\frac{M}{2}-x}\left[p_{M,N-\frac{M}{2}+x}(2)\right]^x\Bigg\}. \end{aligned} \quad (11)$$

The expectations in (4), (9), and (11) are normalized by M to give the corresponding collision rates, and plotted against U in Fig. 1 for $N = 256$ and $M = 2$. Under light load, SF-coded FH-OFDMA with DS1 operates with about two orders of magnitude lower collision rate compared to CH with and without TX diversity, as well as IH without diversity. The performance of DS2 is significantly worse than that of DS1 due to the reduction in $|\tilde{\mathcal{C}}|$. Fig. 1 also depicts that the errors due to approximations in the analytical calculations are very small, and simulations corroborate the theoretical findings. While the performance gap between the hopping strategies narrows rather swiftly (exponentially fast) as the user load increases, and indeed, $\lim_{U\to\infty} E_c(\mathcal{C}, M)/M = \lim_{U\to\infty} E_c(\tilde{\mathcal{C}}, M)/M = \lim_{U\to\infty} E_c^{\mathrm{DS1}}(\tilde{\mathcal{C}}, M)/M = 1$, the margin of improvement delivered by SF coding is still noticeable for $U = 50$.

IV. Simulations and Discussion

The SF block-coding scheme described in Section II is adopted with two TX and one receive (RX) antennas, and no channel coding. The cumulative TX power of the system is kept constant. There are 256 subcarriers in total and $M = 2$. The mobiles transmit quaternary phase-shift keying (QPSK) symbols over frequency-selective finite impulse response (FIR)

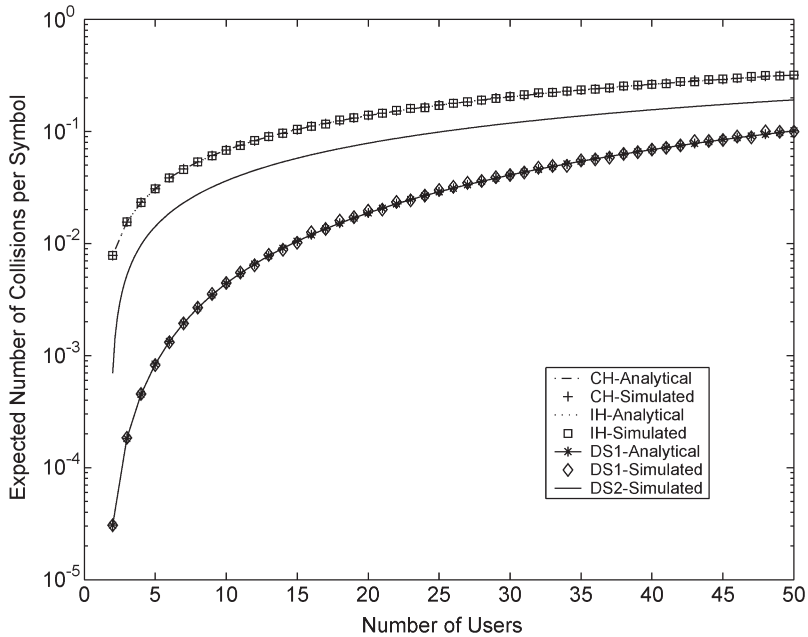

Fig. 1. Analytical and simulated expected number of collisions per symbol in FH-OFDMA with $N = 256$, $M = 2$, and two TX antennas versus the number of active users U for a noiseless, nonfading, MUI-only channel. Without SF coding, IH and CH overlap.

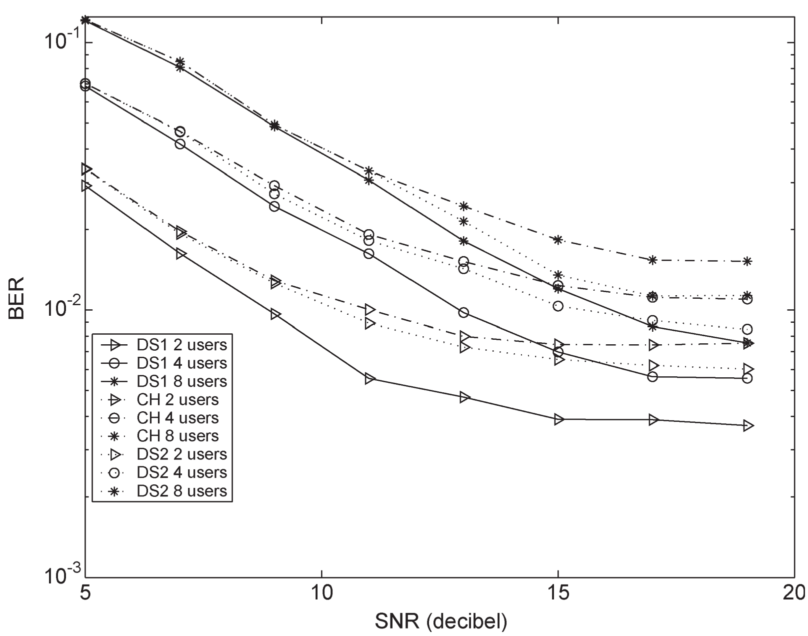

Fig. 2. BER versus SNR (defined with the energy per bit) curves for SF-coded FH-OFDMA with two, four and eight users in frequency-selective fading. (Two TX antennas, $N = 256$, $M = 2$, 2 bits/hop, QPSK modulation.)

channels that are contaminated by additive white complex Gaussian noise. The two uncorrelated FIR channels are of order 8 with complex Gaussian-distributed coefficients. Both channels have average energies set to unity, and the FIR taps are adjusted to correspond to an exponentially decreasing delay spread profile. All signals emanating from all antennas fade independently. The CP length is set to nine symbols. The bit error rate (BER) results reflect the averages of 500 independent channel realizations.

Fig. 2 depicts that at high signal-to-noise ratios (SNRs), the performance-limiting factor is the collision rate. The curves converge to the analytical values that are calculated via (4) and (10) when properly normalized because in the noiseless case $(\mathrm{SNR} \to \infty)$, only MUI, i.e., collisions, account for symbol errors. With more active users in the system, the frequency of collisions increases, inducing a penalty in BER. The BERs for DS1 with eight users and CH with two users are about equal for high SNR [also verified by (4) and (10)], which quantifies the capacity gain offered by IH and SF coding. In Fig. 3, the effect of the number of subcarriers per user M is shown. Comparison of Figs. 1 and 3 indicates that U and M have similar impact in terms of performance loss due to MUI, which is consistent with the analytical expressions developed for the collision rates $E_{\mathrm{c}}(\cdot, M)/M$ and $E_{\mathrm{c}}^{\mathrm{DS1}}(\cdot, M)/M$ in Section III.

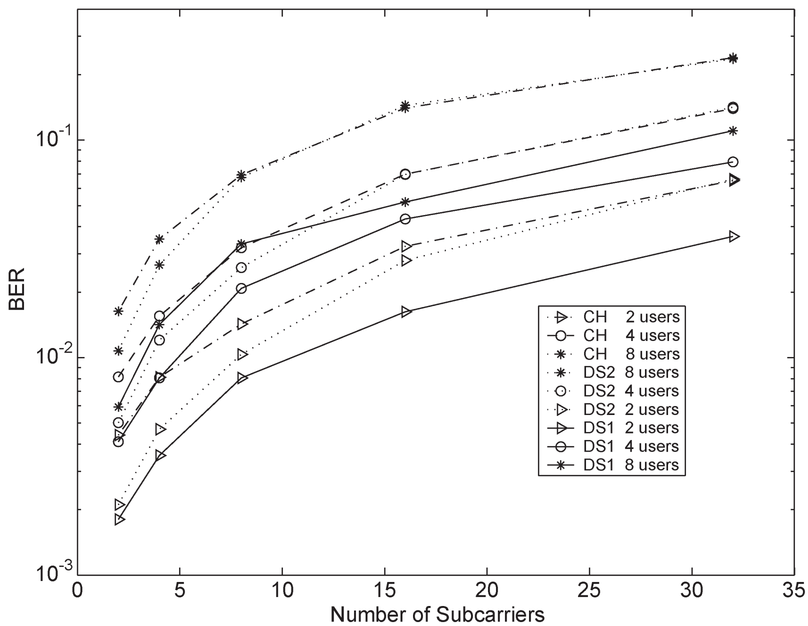

Fig. 3. BER versus number of subcarriers per user M for SF-coded FH-OFDMA in noiseless MUI-only channel. (Two TX antennas, $N = 256, U = \{2, 4, 8\}$, 2 bits/hop, QPSK modulation.)

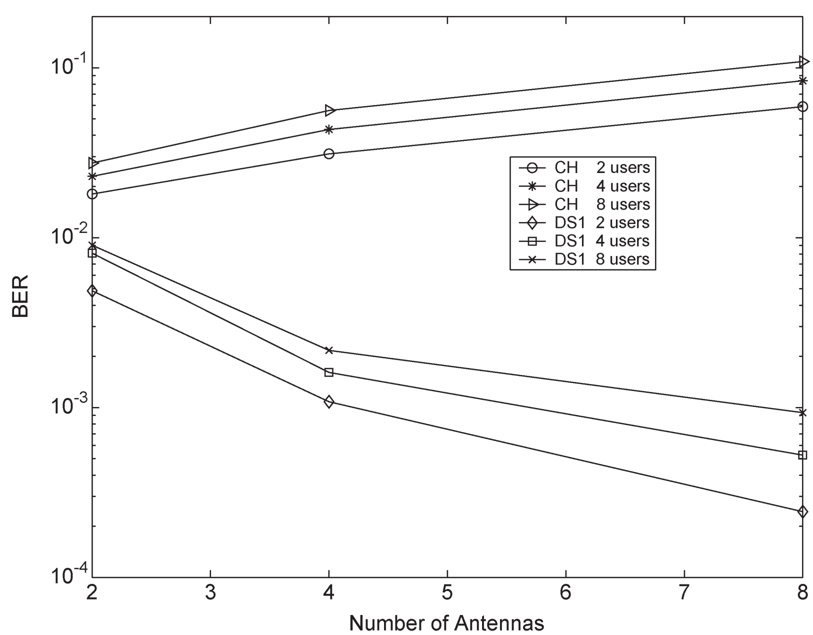

Fig. 4. BER versus the number of TX antennas per user for SF-coded FH-OFDMA in noiseless MUI-only channel. ($N = 256, U = \{2, 4, 8\}$, 2 bits/hop, QPSK modulation, $M = 2$.)

Graphs displaying BER versus the number of TX antennas in Fig. 4 indicate that the performance improves with more antennas for IH, whereas it drops slightly for CH. Here, SF coding based on the orthogonal designs in [10] is employed. Increasing the number of TX antennas means each user occupies more subcarriers per symbol. But then, the effective number of available subcarriers is reduced, which induces a deterioration in CH's BER performance. In other words, the antennas and subcarriers play identical roles for CH, and this also explains the similarity between the curves in Figs. 3 and 4.

With IH, even if only one subcarrier is MUI-free, SF decoding is able to recover the symbol. However, for CH, all subcarriers in a cluster are simultaneously subject to collision, and there is no diversity gain against MUI regardless of how many TX antennas are employed. This bonus, added to the known physical-layer advantages of TX diversity, is unique to SF coding, and it is not attainable through space-time coding alone.

Appendix

Proof of Lemma 1

With M subcarriers per user, let $q_M(z)$ denote the probability that z symbols are collision-free, and define $p_M(x|z)$ as the conditional probability that x symbols collide given that z are known to be collision-free. Consider the case where x symbols (i.e., subcarriers) of a given user are subject to collision, and the remaining $M-x$ symbols are collision-free. The probability that the yth symbol of an interfering user does not collide with the $M-x$ collision-free symbols of the desired user given that the first $y-1$ symbols of the interferer did not collide is $[N-(M-x)-y]/(N-y)$, $y=1,2,\ldots,M$. Then, the probability that $M-x$ symbols are collision-free, with none of the remaining $U-1$ users transmitting over these subcarriers, is

$$q_M(M-x) = \left(\prod_{y=0}^{M-1} \frac{N-(M-x)-y}{N-y}\right)^{U-1}. \quad (12)$$

Next, we calculate the probability that each of the remaining m symbols collide given that the other $M-m$ symbols are collision-free. That is, the rest of the users must select from the $N-(M-m)$ subcarriers. This probability is given by

$$\begin{aligned} p_M(m|M-m) &= \left[1-\left(\prod_{k=0}^{M-1} \frac{N-(M-m)-k-1}{N-(M-m)-k}\right)^{U-1}\right]^m \\ &= \left[1-\left(\frac{N-2M+m}{N-M+m}\right)^{U-1}\right]^m \quad (13) \end{aligned}$$

assuming that each symbol-collision event is independent of the other symbols' collisions.

Multiplying (12) and (13) by the number of possible combinations yields the probability of an m-bit collision for one user, $p_M(x) = \binom{M}{x} p_M(x|M-x) q_M(M-x)$. The expected number of symbols involved in a collision for IH is then $E_c(\tilde{\mathcal{C}}, M) = \sum_{x=1}^{M} x p_M(x)$. ■

Proof of Lemma 2

Given the definition of $p_{M,L}(x)$ in the Lemma, we have

$$p_{M,L}(0) = \left[\frac{(L-M)(L-M-1)}{L(L-1)}\right]^{U-1}$$

$$p_{M,L}(1) = 2\left(\frac{L-M}{L}\right)^{U-1}\left[1-\left(\frac{L-M-1}{L-1}\right)^{U-1}\right]$$

$$p_{M,L}(2) = 1 - p_{M,L}(0) - p_{M,L}(1).$$

Let y count the number of collision-free subcarrier pairs used in SF coding. There are $\sum_{x=1}^{M/2} 2x\binom{M/2}{x}$ possible scenarios where x out of the $M/2$ pairs experience a double subcarrier collision. Given x double collisions, the remaining $M/2-x$ pairs are either collision-free, or subject to a single collision. There are $\sum_{y=1}^{M/2-x}\binom{M/2-x}{y}[p_{M,N-2y}(1)]^{M/2-x-y}$ possible cases of single collisions.

Next, $[p_{M,N-y-M/2+x}(2)]^x$ is the probability of x double collisions given that there are y noncolliding subcarrier pairs, and $\prod_{z=0}^{y-1} p_{M,N-2z}(0)$ corresponds to the probability that y pairs are collision-free. Recalling that $2x$ symbols collide in x double collisions, and putting all the above calculations together finally yields (11). Note that for index-numbering purposes, the last term, which corresponds to $y=0$, is formulated separately. ■

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