SPACE-TIME CODING AND SIGNAL SPACE DIVERSITY IN THE PRESENCE OF CHANNEL ESTIMATION ERRORS

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Abstract

Joint space-time (ST) coding and constellation rotation is considered in the presence of imperfect channel state information. The combined system with two transmit antennas is shown to be more resistant to channel estimation errors than ST coding. Experiments involving several cases of channel estimation errors verify related theory reported in the literature.

Keywords: *Space-time coding, signal space diversity, channel estimation error, transmit diversity*

1. INTRODUCTION

A well-known approach for increasing link quality against multipath fading is to apply receiver diversity. For wireless communication systems where it is not feasible to deploy multiple antennas at the receiver, transmitter diversity offers a powerful alternative. Many techniques incorporating space-time (ST) coding have been proposed in recent years (see e.g. [8, 9] and the references therein). In [1] and [11] orthogonal designs achieving full diversity are presented. However, these orthogonal designs exist only for the cases where number of antennas is equal to $2, 4$ and 8, and rate-1 codes exist uniquely only for two antennas.

Alternatively, signal space diversity combats fading by rotating the constellation and applying componentwise interleaving [2]. The interleaver is required for the independent fading of the real and imaginary parts of the constellation points. Although this is a bandwidth and power efficient solution, it introduces additional complexity and time delay as a result of the interleaving procedure.

Recently, another model that uses both signal space diversity and ST coding is introduced in order to achieve full diversity for more than two antennas, while simultaneously eliminating the interleaver from the system design [5, 12]. In [5], no extra gain is achieved from the introduction of rotated constellations with two antennas, which is expected because the Alamouti scheme [1] exploits all the diversity in the system.

The impact of channel estimation errors on the system performance has been investigated for transmit diversity with ST coding. Specifically, the criteria that are customarily used in code design for coherent receivers do not induce any penalty in the diversity order when the channel state information (CSI) is imperfect, even though the bit error rate may suffer [6, 10]. This is intuitively plausible because diversity gains become observable at high signal-to-noise ratios (SNR), where estimates are very reliable [6]. Bit error probability computations and formulas for a complex Gaussian error distribution are presented in [3, 4].

In all relevant work [2, 5, 12], the channel is assumed to be perfectly known and constant throughout the code block. In this paper, we present joint ST coding and signal space diversity for two transmit antennas, and show that the combined system is more resistant to channel estimation errors, compared to ST coding alone.

The organization of the paper is as follows: In Section II, we present the system model which employs a simple scheme that combines the diversity techniques of [1] and [2]. We test the performance of the joint diversity system for different estimation error types in Section III, and follow that with the conclusions in Section IV.

2. SYSTEM MODEL

We consider a hybrid system that combines the diversity ideas in [1] and [2] (hereafter referred to as joint diversity). The system architecture is given in Fig. 1. Let a_0 and a_1 be two rotated complex constellation points that are consecutively transmitted. For simplicity, we assume $\pi/8$-rotated quadrature phase-shift keying (QPSK) symbols [2]. With

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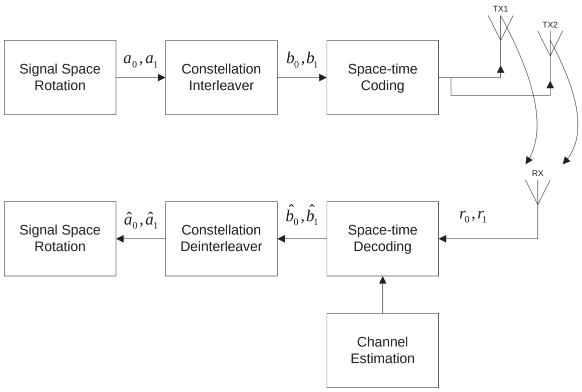

Fig. 1. The system model for combined space-time coding and signal space diversity.

constellation interleaving, the real and complex parts of the symbols are interchanged so that

$$b_0 = \Re\{a_0\} + i\Im\{a_1\},$$

$$b_1 = \Re\{a_1\} + i\Im\{a_0\},$$

$i = \sqrt{-1}$ are obtained. Next, the symbols b_0, b_1 are space-time coded according to the Alamouti scheme, and eventually transmitted through the two transmit antennas..

The received baseband signals are

$$\begin{aligned} r_0 &= h_0 b_0 + h_1 b_1 + n_0, \\ r_1 &= -h_0 b_1^* + h_1 b_0^* + n_1, \end{aligned} \tag{1}$$

where h_0, h_1 and n_0, n_1 are the complex channel coefficients and the complex additive white Gaussian noise for antennas 1 and 2, respectively. Space-time processing via Alamouti-type decoding of r_0, r_1 and component-deinterleaving achieve maximum diversity gain as in [1] with the availability of CSI. To account for channel estimation errors, consider the channel matrix H as

$$\mathbf{H} = \begin{vmatrix} h_0 + \xi_0 & h_1 + \xi_1 \\ h_0 + \xi_2 & h_1 + \xi_3 \end{vmatrix} \tag{2}$$

where the columns of **H** correspond to channels seen by different antennas, and rows correspond to channel consitions at different time instants. The estimation errors $\{\xi_i\}_{i=0}^3$ are modeled as random variables with zero-mean additive, complex white Gaussian distributions and variances $\{\sigma_i^2\}_{i=0}^3$, respectively. Hence, (1) becomes

$$\begin{aligned} r_0 &= (h_0 + \xi_0) b_0 + (h_1 + \xi_1) b_1 + n_0 \\ r_1 &= -(h_0 + \xi_2) b_1^* + (h_1 + \xi_3) b_0^* + n_1. \end{aligned} \tag{3}$$

The zero-mean assumption ensures that the estimates are unbiased, which is necessary to preserve the diversity order regardless of the CSI imperfections [6]. The optimal combining of [1] gives

$$\begin{aligned} \hat{b}_0 &= (|h_0|^2 + |h_1|^2) b_0 + I_0 + h_0^* n_0 + h_1 n_1^* \\ \hat{b}_1 &= (|h_0|^2 + |h_1|^2) b_1 + I_1 + h_1^* n_0 - h_0 n_1^* \end{aligned} \tag{4}$$

where the interference terms I_0, I_1 are

$$\begin{aligned} I_0 &= h_1^*(\xi_0 b_0 + \xi_1 b_1) + h_0(\xi_2^* b_1 + \xi_3^* b_0) \\ I_1 &= h_1(\xi_3^* b_0 + \xi_2^* b_1) + h_0^*(\xi_1 b_1 - \xi_0 b_0) \end{aligned} \tag{5}$$

For high SNR, the interference terms in (5) are the performance limiting factors, rather than the noise components in (4). On the other hand, if $h_0, h_1, \xi_0, \xi_1, \xi_2$ and ξ_3 are all mutually uncorrelated as per the independent fading assumption [10], then so are I_0 and I_1. This fact can be exploited through signal space diversity. If component interchange is again applied for the symbols at the receiver as

$$\hat{a}_0 = \Re\{\hat{b}_0\} + i\Im\{\hat{b}_1\}, \hat{a}_1 = \Re\{\hat{b}_1\} + i\Im\{\hat{b}_0\},$$

then both the real and the imaginary parts of the symbols are contaminated by random but uncorrelated interference, which results in a system that is more tolerant to channel estimation errors. In the next section we support this idea via simulation results.

3. SIMULATIONS

The simulations are performed for the system model given in the previous section for 1000 independent channel realizations. Each run is terminated when 1000 errors are observed. The transmitter sends $\pi/8$-rotated QPSK symbols as in Fig. 2. The complex Gaussian distributed channel gain has unity variance. Several channel estimation error scenarios are considered. When there is no estimation error,

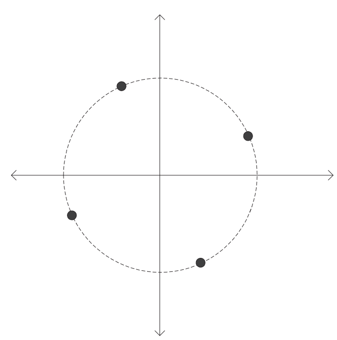

Fig. 2. The $\pi/8$-rotated QPSK constellation [2].

the results in [5] and our simulations both indicate that the performance is the same for the Alamouti scheme with or without signal space diversity. Hence, in the bit error rate

(BER) graphs, we refer to both with a single line labeled as '"no estimation error".

In Fig. 3, the effect of estimation error in only one channel coefficient is considered. The error variance is taken as 0.01, and the channel gain is assumed to be unity. Simulations show that the system performance does not depend on which specific error ξ_i is set to be nonzero. Fig. 3 indicates that the BER performance of joint diversity is very close to that of the Alamouti scheme with perfect CSI. For $E_b/N_0 > 5$ dB, joint diversity it is uniformly better than the Alamouti scheme alone when the latter operates in the presence of channel estimation errors.

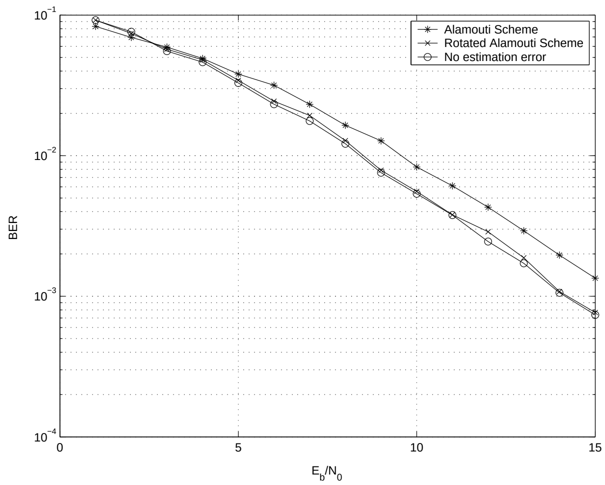

Fig. 3. Two-antenna transmit diversity performance when only one channel estimation is incorrect.

Next, we assume that errors exist in the second time slots only; i.e., $\sigma_0^2 = \sigma_1^2 = 0, \sigma_2^2 = \sigma_3^2 = 0.01$. This is a realistic scenario because channel estimation errors are more likely to happen in the second time slot due to the variations in the channel coefficients over time. The gain of the joint diversity scheme is given in Fig. 4, which exhibits the same slopes as those in Fig. (3). Thus, diversity orders are preserved even if the CSI is imperfect, which is consistent with the theory reported in [6] and [10].

The gain of the joint diversity for the case when $\sigma_0^2 = \sigma_1^2 = \sigma_2^2 = \sigma_3^2 = 0.01$ is displayed in Fig. 5. Unlike the previous experiments, the bit error rate deteriorates slightly here due to the back-to-back erroneous estimates.

Finally, we evaluate the impact of the variance of the error random variables. The BER performances under the parameter sets $\sigma_0^2 = \sigma_1^2 = 0, \sigma_2^2 = \sigma_3^2 = 0.02$ and $\sigma_0^2 = \sigma_1^2 = 0, \sigma_2^2 = \sigma_3^2 = 0.04$ are presented respectively in Fig. 6 and Fig. 7. The performance of combined ST coding and constellation rotation degrades with increased error variance, approaching that of space-time coding alone.

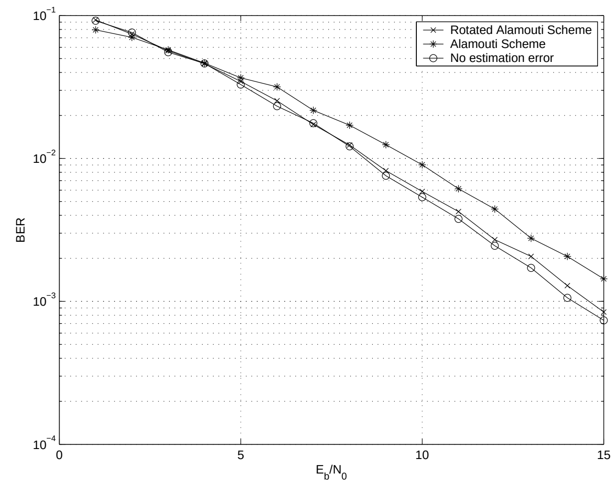

Fig. 4. The effect of channel estimation errors on the transmit diversity schemes for $\sigma_0^2 = \sigma_1^2 = 0, \sigma_2^2 = \sigma_3^2 = 0.01$.

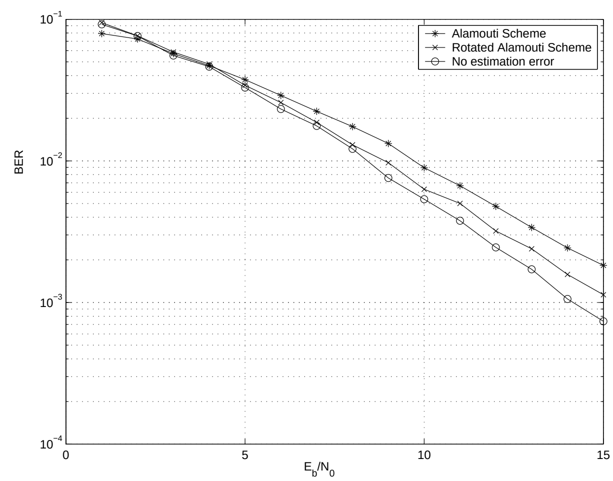

Fig. 5. The effect of channel estimation errors on the transmit diversity schemes for $\sigma_0^2 = \sigma_1^2 = \sigma_2^2 = \sigma_3^2 = 0.01$.

For all the cases presented above, the BER performance offered by space-time coding improves when it is employed in conjunction with signal space diversity described in the form of constellation rotation, even though there is no appreciable gain in diversity order. While there is no BER gain up to $E_b/N_0 = 5$ dB, the improvement becomes noticeable around $E_b/N_0 = 15$ dB, when the diversity gain kicks in and estimation errors are less pronounced.

4. CONCLUSIONS

In this paper, we considered joint space-time coding of [1] and signal space diversity of [2]. Even though constellation rotation on top of the Alamouti scheme does not offer

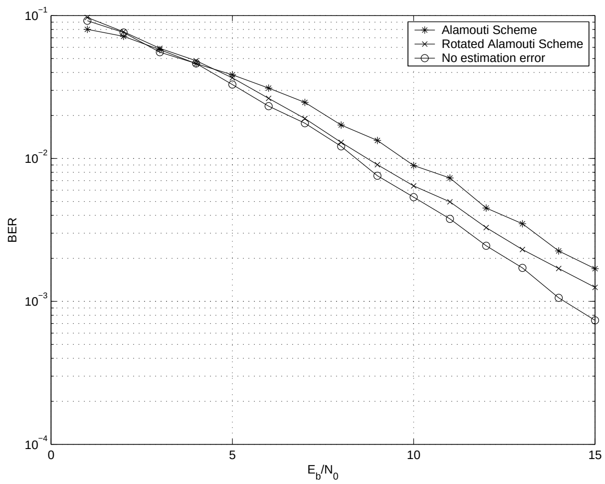

Fig. 6. The effect of channel estimation errors on the transmit diversity schemes for $\sigma_0^2 = \sigma_1^2 = 0, \sigma_2^2 = \sigma_3^2 = 0.02$.

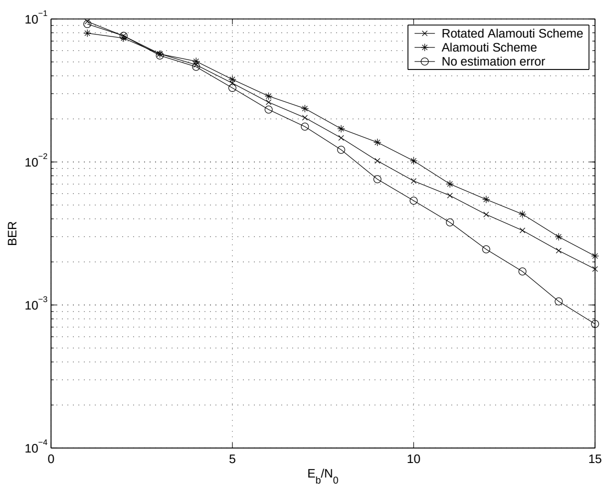

Fig. 7. The effect of channel estimation error on the transmit diversity schemes when $\sigma_0 = \sigma_1 = 0, \sigma_2 = \sigma_3 = 0.04$

additional diversity gain for two antennas, the combination is very robust to CSI imperfections, which may be worth the incurred complexity. Under unbiased channel estimates with variances comparable to those of the channel gains, the diversity order is verified to be independent of estimation errors.

In [7], a rule of thumb is prescribed where the CSI can be regarded as near-perfect so long as the error variance is negligible compared to the reciprocal of the SNR. This rule is satisfied in the experiments above for the $E_b/N_0 = 10 - 15$ dB range, and the results are in agreement with [7], particularly for the joint diversity system.

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